EFFECT OF A GAS STREAM ON THE WAVE FLOW OF THIN LAYERS OF A VISCOUS LIQUID

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An analytical study was undertaken of the effect of a gas stream on the wave flow characteristics of thin layers of a viscous liquid moving over a vertical surface. The results of the investigations are compared with experimental data.

Following [1, 2] on the wave flow of thin layers of a viscous liquid over a vertical surface, a number of investigations appeared in the literature extending this topic and directed at the refinement of the equations of flow of liquid films in a motionless gaseous medium under the action of gravitational forces [3, 4], at the development of more accurate methods of solving these equations [3, 5], and at the determination of the stability of laminar flow of a film [6]. In [7], the effect of the tangential stresses created at the interface on the wavelength is considered.

In order to investigate the effect of the gas flow on the wave characteristics of thin layers of a viscous liquid moving over a vertical surface, we shall use the procedure of [1] in general form. We orient the axis x along the wall in the direction of action of the force of gravity and the axis y from the wall in the direction of the liquid. We denote the variable thickness of the liquid layer by $a(x, t)$ and its average thickness by a_0. Suppose that

$$a = a_0(1 + \varphi), \quad \varphi = \varphi(x, t). \tag{1}$$

We shall assume the wavelength to be considerably greater than the thickness of the liquid layer ($\lambda \gg a_0$). Calculations by the formulas $\lambda = 2\pi n^{-1}$, Eqs. (15) and (14) and by the graphs (Fig. 1) show that up to values of $M < 100$ the relation $\lambda a_0^{-1} > 10$ holds for almost all gas–liquid flows.

The motion of a liquid film is described by the Navier–Stokes conditions which, for the condition $\lambda > a_0$ are converted to the form

$$\frac{\partial u_x}{\partial t} + u_x \frac{\partial u_x}{\partial x} + u_y \frac{\partial u_x}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + g + \nu \frac{\partial^2 u_x}{\partial y^2}, \tag{2}$$

of continuity

$$\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} = 0 \tag{3}$$

and material balance of the liquid film

$$\frac{\partial ua}{\partial x} + \frac{\partial a}{\partial t} = 0, \quad u = \frac{1}{a} \int_0^a u_x dy \tag{4}$$

with the following boundary conditions

$$u_x = u_y = 0 \ \text{for} \ y = 0,$$
$$p = -\sigma \frac{\partial^2 a}{\partial x^2} + p_0, \ \tau_1 = \pm\tau \ \text{for} \ y = a. \tag{5}$$

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The latter condition arises from [7] when $\lambda > a_0$. In these expressions p_0 is the pressure in the gas; τ_1 and τ are the tangential stresses at the interface respectively from the direction of the liquid and gas. According to [1], we take

$$\tau = \frac{\gamma}{\pi}(a_0 \alpha n)^2 (v - k)^2 \rho_2, \quad \gamma \leqslant 1. \tag{6}$$

Assuming that φ is a function of $x - kt$, i.e., that the profile of the surface of separation is moving with constant velocity k, we have

$$k\frac{\partial a}{\partial x} = -\frac{\partial a}{\partial t}, \quad \frac{\partial u_x}{\partial t} = -k\frac{\partial u_x}{\partial x}. \tag{7}$$

From the first equation of (7), (4), and (1), taking into account that $Q = \overline{ua}$ (the bar denotes the average with respect to x), it is not difficult to obtain

$$Q = u_0 a_0, \quad u = u_0(1 + \varphi z)(1 + \varphi)^{-1}. \tag{8}$$

Using Eqs. (3) and (5) and the second equation of (7), we represent Eq. (2) in the form

$$(u_x - k)\frac{\partial u_x}{\partial x} - \frac{\partial u_x}{\partial y}\int \frac{\partial u_x}{\partial x}\,dy = \frac{\sigma}{\rho}\frac{\partial^3 a}{\partial x^3} + g + \nu\frac{\partial^2 u_x}{\partial y^2}. \tag{9}$$

We substitute Eq. (9) by an equation integrated with respect to y. We put

$$u_x = 3u(x, t)\left(\frac{y}{a} - \frac{y^2}{2a^2}\right) - \frac{\tau a}{4\mu}\left(2\frac{y}{a} - 3\frac{y^2}{a^2}\right), \tag{10}$$

which coincides with the accurate solution for laminar flow of a layer of liquid with the boundary conditions $u_x = 0$ when $y = 0$ and $\mu \partial u_x/\partial y = \tau$ when $y = a$. Expression (10) can be written as the first step in the successive use of a direct method for determining u_x from Eqs. (2), (3), and the boundary conditions (5).

Substituting u_x from Eq. (10) in Eq. (9) and integrating with respect to y from 0 to a, we obtain

$$\left(0.9u - \frac{\tau a}{5\mu} - k\right)\frac{\partial u}{\partial x} = \frac{\sigma}{\rho}\frac{\partial^3 a}{\partial x^3} + g + \nu\left(\frac{3\tau}{2\mu a} - \frac{3u}{a^2}\right). \tag{11}$$

When calculating the coefficients of this equation, the quantity a, just as in [1], was assumed to be constant. We introduce a from Eq. (1) and u from Eq. (8) into Eq. (11) and, limiting to a first approximation, we find

$$\frac{\sigma}{\rho}a_0\varphi''' + u_0(z-1)\left(z - 0.9 + \frac{\tau a_0}{5\mu u_0}\right)\varphi' + 3\left(g - z\frac{u_0\nu}{a_0^2} + \frac{\tau}{\rho a_0}\right)\varphi + \left(g - 3\frac{u_0\varphi}{a_0^2} + \frac{3\tau}{2\rho a_0}\right) = 0. \tag{12}$$

If $\varphi = 0$, then plane laminar flow occurs. If we denote the thickness of the liquid film in this flow by m and take account of Q from Eq. (8), we find from Eq. (12)

$$g = 3Q\nu m^{-3} - 1.5\tau\rho^{-1}m^{-1}. \tag{13}$$

For the existence of a stable periodic solution it is necessary that in Eq. (12) the free term and the coefficient for φ should be equal to zero. Hence, it follows that Eq. (13) should be satisfied and, moreover,

$$g a_0^3 \nu^{-1} Q^{-1} = z - T. \tag{14}$$

In satisfying these conditions, the periodic stable solution of Eq. (12) is determined in the form

$$\varphi = \alpha \sin\left[(x - kt)n\right],$$

and the wave number

$$n^2 = \rho\sigma^{-1}a_0^{-1}u_0^2(z-1)(z - 0.9 + 0.2T). \tag{15}$$

We denote by $\beta = a_0 m^{-1}$ the ratio of the corresponding thicknesses of liquid layers with identical flow rates Q; then from Eqs. (13) and (14)

$$T = \frac{6\beta^3 - 2z}{3\beta - 2}. \tag{16}$$

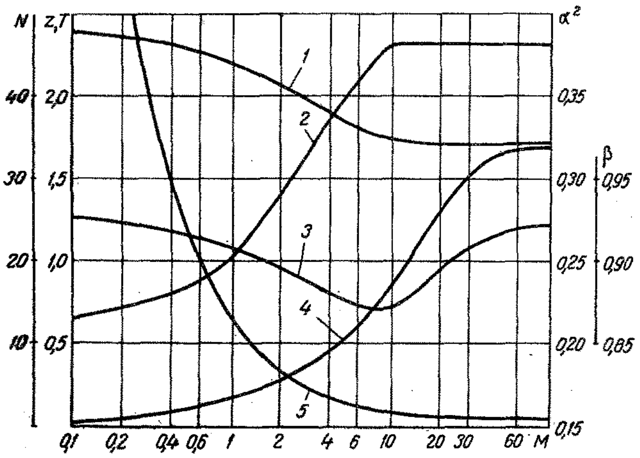

Fig. 1. Dependence of z, α^2, β, N, and T on M: 1) z; 2) α^2; 3) β; 4) T; 5) N.

We determine the dissipation of energy through the transverse cross section of the flow by an element of length dx [8]:

$$dE_\mu = -\mu dx \int_0^a \left(\frac{\partial u_x}{\partial y}\right)^2 dy.$$

Substituting u_x from Eq. (10) in this equation, and averaging the expression obtained over the wavelength, we find – taking account of Eqs. (8) and (1) – the average energy dissipation per unit length

$$-E_\mu = 3\mu Q^2 a_0^{-3}\left(F + \frac{1}{12} T^2\right), \tag{17}$$

where, according to [1],

$$F = \frac{1}{2}\{2 + \alpha^2[1 - 6z + z^2(1 + 2\alpha^2)]\}(1 - \alpha^2)^{-\frac{5}{2}}. \tag{18}$$

If $\alpha = 0\,(F = 1)$ and $\tau = 0\,(T = 0)$, the quantity $-E_\mu$ from Eq. (17) becomes equal to the energy dissipation of the liquid in the case of plane laminar flow. We denote by $E = -[(E_\mu a_0^3 \mu^{-1} Q^{-2})/3]$ the relative energy of dissipation; then according to Eq. (17)

$$E = F + 1/12T^2. \tag{19}$$

In the case of interaction of the liquid with the gas stream, the energy of dissipation (17) is compensated by the work of the body forces $g\rho Q$ and the energy imparted by the gas stream to the surface of the liquid τk, i.e.,

$$3\mu Q^2 a_0^{-3}(F + 1/12T^2) = g\rho Q + \tau k. \tag{20}$$

Solving jointly this equation and Eq. (13), we find

$$F = \beta^3 - 1/12T(6\beta + T - 4z). \tag{21}$$

Equations (12)-(17) and (19)-(21) in the case when the volocity of the gas is zero ($\tau = 0$, $T = 0$) are transformed into the corresponding equations obtained in [1].

Eliminating the quantity n from Eq. (6) by substitution of its value from Eq. (15) we find

$$T = \frac{5\alpha^2(z-1)(z-0.9)}{N - \alpha^2(z-1)}; \quad N = \frac{5\mu\sigma\pi}{\gamma\rho\rho_2 a_0 (v-k)^2}. \tag{22}$$

In order to determine the wave parameters, we shall use the reasoning that as a result of wave flow the amplitude of the wave will be increased until the energy of dissipation reaches the permissible minimum value and the thickness of the liquid is reduced to the value determined by Eq. (20). From the condition for stability of the periodic solution, Eqs. (13) and (14) also should be taken into account. In our case, these

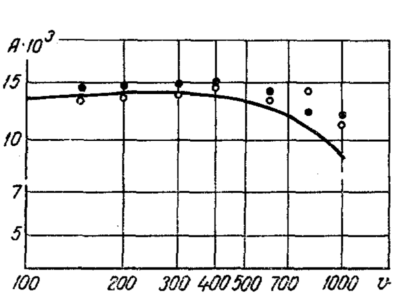

Fig. 2

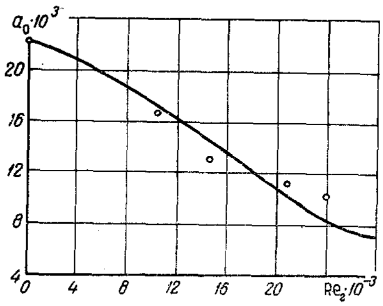

Fig. 3

Fig. 2. Effect of gas flow rate (v, cm/sec) on the wave amplitude of the water film ($A \cdot 10^3$ cm), for Re = 100: points are experimental data from [9]; the curve is calculated by formulas (25), (28), and (27).

Fig. 3. Dependence of the average thickness of the water film ($a_0 \cdot 10^3$ cm) on Reynold's number for a gas with Re = 35: points are experimental data from [10]; the curve is calculated by formulas (25), (26), and (28).

considerations are effected in the following way. For brevity we shall denote the quantity T determined by Eq. (22) by $T(\alpha, z, N)$ and determined by Eq. (16) by $T(\beta, z)$; the quantity F from Eq. (18) is denoted as $F(\alpha, z)$ and from Eq. (21) as $F(\beta, z)$ by the substitution of $T(\beta, z)$. In addition, from Eq. (19) we denote

$$E(\alpha, z, N) = F(\alpha, z) + \frac{1}{12} T^2(\alpha, z, N),$$

$$E(\beta, z) = F(\beta, z) + \frac{1}{12} T^2(\beta, z).$$

Then the conditions for determining the wave parameters are written in the form

$$T(\beta, z) = T(\alpha, z, N); \; E(\alpha, z, N) = E(\beta, z); \quad \frac{\partial E(\alpha, z, N)}{\partial \alpha^2} = 0.$$

As the values of T which occur in the left and right hand sides of the second equation of this system are equal, then this equation is simplified. Finally, we obtain

$$T(\beta, z) = T(\alpha, z, N); \; F(\alpha, z) = F(\beta, z); \quad \frac{\partial E(\alpha, z, N)}{\partial \alpha^2} = 0. \tag{23}$$

In the case of a stationary gas $v - k = 0$, $N = \infty$, and $T(\alpha, z, N) = 0$, we obtain from Eq. (23) the system of equations

$$z = 3\beta^3, \quad F(\alpha, z) = \beta^3; \quad \frac{\partial F(\alpha, z)}{\partial \alpha^2} = 0,$$

for which, in [1], were found

$$\alpha^2 = 0.21; \quad z = 2.4; \quad F = 0.8; \quad \beta = 0.93. \tag{24}$$

Change of the wave flow parameters should be found as a function of the quantity in which v and Q occur but in which a_0 is absent. The required value can be obtained by substitution of $z - T$ from Eq. (14) and N from Eq. (22) in the expression

$$M = \frac{5\pi}{(z - T)^{1/3} N} = \frac{\gamma \rho_2 \nu^{2/3} \mathrm{Re}^{4/3} (v - k)^2}{g^{1/3} \sigma}. \tag{25}$$

The system of equations (23) is solved on a computer. The results of the calculation are shown graphically in Fig. 1. When M = 0 (T = 0) the value of all quantities coincides with Eq. (24).

From Eq. (14), and also taking into account that $A = \alpha a_0$ and $k = zu_0 = z\nu \mathrm{Re} a_0^{-1}$, we obtain

$$a_0 = g^{-1/3} \nu^{2/3} (z - T)^{1/3} \mathrm{Re}^{1/3}; \tag{26}$$

$$A = g^{-1/3} \nu^{2/3} \alpha (z - T)^{1/3} \mathrm{Re}^{1/3}; \quad (27)$$

$$k = g^{1/3} \nu^{1/3} z (z - T)^{-1/3} \mathrm{Re}^{2/3}. \quad (28)$$

The experimental values of amplitude are plotted in Fig. 2 as a function of the air flow rate, measured over two sections along the tube height [9] with constant water flow rate (Re = 100). The theoretical curve is calculated in this sequence: v – k given by Eq. (25), M calculated, α, z, and T determined from Fig. 1, k by Eq. (28), A by Eq. (27), and then v = (v – k) + k.

The experimental values of water film thickness are plotted in Fig. 3, measured at a constant water flow rate (Re = 35) and different values of Reynold's number for air ($\mathrm{Re}_g = vd\nu_g^{-1}$) for a descending movement of water and air along a vertical tube with a diameter of 9.87 mm [10]. The theoretical curve is calculated by formulas (25), (26), (28), and Fig. 1. It was assumed in the calculations that $\gamma = 1$.

NOTATION

a_0	is the average thickness of liquid layer;
u	is the average velocity in section;
u_0	is the mean rate of flow of liquid in center section;
k	is the phase velocity of motion of interface profile;
$z = ku_0^{-1}$;	
v	is the average flow rate of gas;
A and λ	are the amplitude and wavelength;
$\alpha = Aa_0^{-1}$;	
ν and μ	are the coefficients of kinematic and dynamic viscosity of the liquid;
ρ and ρ_2	are the densities of liquid and gas;
$\mathrm{Re} = a_0 u_0 \nu^{-1} = Q\nu^{-1}$	is the Reynold's number of the liquid film;
$T = \tau a_0 \mu^{-1} Q^{-1}$;	
M and N	are given in the text by Eq. (25) and (22).

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